MINI PAPER

Sample Sizes and Marketing Campaigns

by Vivek Ajmani

Introduction

Marketing campaigns often involve testing the effectiveness of different treatments with respect to some response of interest. For instance, a credit card company may be interested in determining whether changing the design of an information brochure increases the response rate in a direct mail study. The company may decide to test the effectiveness of the 'new creative' by mailing out the standard brochure and the new brochure. The response rates from the two mailings may be compared to gauge the effectiveness of the new treatment. Choosing the correct sample size, therefore, is important to minimize the chances of an incorrect decision.

Sample size determination is a key component and a preliminary step in most studies involving data collection and analysis. Incorrect sample sizes often lead to incorrect decisions. For instance, small sample sizes may not allow us to detect true differences between the treatments of interest. On the other hand, large sample sizes may detect small differences which are of no practical interest as far as business decisions are concerned.

This paper discusses the two commonly used methods for sample size determination in marketing studies and provides guidelines on selecting the correct method based on business objectives.

Type 1 and Type 2 Errors

Consider the example where a credit card company wants to use a direct mail study to test a new creative (A) versus the existing creative (B, the control) with respect to the response rate. Let p_A and p_B be the true response rates of the creative's. The credit card company will select the new creative if its response rate is significantly higher than the response rate of the existing creative. Consider then, conducting the hypothesis test $H_0 : p_A \leq p_B$ versus $H_1 : p_A > p_B$.

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A Type 1 error occurs if the null hypothesis is rejected when it is true. That is, if we claim that the response rate of the new creative is significantly higher than the response rate of the existing creative when in fact it is not. A business consequence of this decision may be increased costs if the new creative is used and if it is costlier than the existing creative. A Type 2 error occurs if the null hypothesis is not rejected when it should be rejected. That is, if we claim that the response rate of the new creative is no different from (or worse than) the response rate of the existing creative when in fact it is better. A business consequence of this decision may be the missed opportunities to increase the client base. Sample sizes are chosen to minimize the Type 1 and Type 2 errors.

The Confidence Interval Approach to Sample Size Calculations

The large sample $(1-\alpha)100\%$ confidence interval for the true difference in the response rates $p_A - p_B$ is given by

$$(\hat{p}_A - \hat{p}_B) \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n_A} + \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}}$$

where $\hat{p}_A$ and $\hat{p}_B$ are the observed response rates from sample sizes n_A and n_B, respectively and $Z_{\alpha/2}$ is the Z-score associated with the $\alpha/2$ percentile of the standard normal distribution. The sample sizes are calculated by specifying a bound on the margin of error term. If we let B be the upper bound on the margin of error, then

$$Z_{\alpha/2}\sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n_A} + \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}} \leq B.$$

It can be shown that

$$n_A \geq \frac{\hat{p}_A(1-\hat{p}_A)}{\left(\frac{B}{Z_{\alpha/2}}\right)^2 - \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}}.$$

Therefore, the sample size for the new creative can be calculated by specifying values for $\hat{p}_A, \hat{p}_B, B, \alpha$, and n_B. Here, $\hat{p}_A$ and $\hat{p}_B$ are simply the expected response rates for the two treatments.

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It is important to note that this approach does not use the Type 2 error in the computations. It can be shown that this approach to calculating sample sizes results in a Type 2 error of approximately 50%. That is, there is a 50% chance of detecting a true difference between the response rates of the two treatments.

Sample Sizes Based on Hypothesis Testing

Sample sizes calculated using the hypothesis testing approach make use of the Type 2 error and therefore allows the experimenter a greater flexibility in optimizing the sample sizes to minimize both types of errors. It can be shown that the sample size associated with the new creative is given by

$$n_A = p_A(1-p_A)\left[\frac{\delta^2}{(Z_\alpha - Z_{1-\beta})^2} - \frac{p_B(1-p_B)}{n_B}\right]^{-1}.$$

Here,

n_A is the sample size of the experimental group

p_A is the expected response rate of the experimental group

p_B is the expected response rate of the control group

δ is the difference between p_A and p_B that needs to be detected

n_A is the sample size of the experimental group

Z_α is the Z score associated with the Type 1 error

$Z_{1-\beta}$ is the Z score associated with the Type 2 error

Selecting the Sample Size Calculation Method

The method used to calculate the sample sizes depends upon the business objectives associated with the study. In the credit card example, if the cost associated with the two treatments are comparable then the company may simply want to know whether the response rate of the new treatment is better than the response rate of the control treatment. The sample sizes can therefore be calculated by using the confidence interval method. However, if the cost associated with the new treatment is higher than the cost associated with the

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control treatment then the company may want the response rate of the new treatment to be higher than the response rate of the control treatment by a certain amount (δ) in order to offset the higher cost. In this case, the sample sizes should be calculated by using the hypothesis testing method.

Example

To illustrate the two methods, consider the credit card example again. Assume that the expected response rates for the two treatments are 6% and 5%. That is, a difference of 1%. Assume that the bound on the error desired is 1% also. Furthermore, assume that the Type 1 and Type 2 errors are 5% and 10%, respectively. Also assume that the sample size for the control treatment is restricted at 10,000. The sample size for the new treatment using the confidence interval approach is given by

$$n_A \geq \frac{0.06\times 0.94}{\left(\frac{0.01}{1.96}\right)^2 - \frac{0.05\times 0.95}{10000}} = 2651.$$

The sample size using the hypothesis test approach is given by

$$n_A = 0.06\times 0.94\left[\frac{0.01^2}{(-1.645-1.28)^2} - \frac{0.05\times 0.95}{10000}\right]^{-1} = 8129.$$

The sample size calculated using the confidence interval approach is smaller because of the high Type 2 error (50%). Here, if the observed difference is 1% then there is a 50% chance that the true difference is greater than or less than 1%. As mentioned earlier, if the costs associated with the two treatments are comparable then the company may not care by how much the response rate of the new treatment is better as long as it is better. On the other hand, if the cost associated with the new treatment is higher then the company may want to see the response rate of the new treatment higher than the response rate of the control treatment by a certain amount.