Process Improvement through simplex EVOP

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Small process improvements can lead to important dollar gains using evolutionary operation (EVOP). As I indicated in my last column (1), EVOP involves deliberately and methodically, perturbing the controlled variables of a process in a search for improved performance.

The first column described the EVOP concept and the classical EVOP approach. This column deals with a useful variation of the classical approach, known as simplex EVOP. Simplex EVOP is especially suitable when

- The process variables can be perturbed only at special times, such as at shift change
- There are a number of product performance criteria
- More than 2 or 3 process variables are to be perturbed
- Process performance is changing over time
- Statistical calculations need be minimized

Basic concept

Simplex EVOP works as follows. First, the process performance characteristic(s) to be improved are defined. The process variables whose perturbation might lead to performance improvement are then identified and their present conditions noted. Next, initial perturbation steps away from the present condition are selected for each of the identified process variables; these steps must be sufficiently small so that serious deleterious effects resulting from the perturbation are unlikely.

The conditions for an initial series of runs is laid out as the corners of a simplex (triangle for two variables, tetrahedron for three variables) involving the process variables. For example, if two process variables are to be perturbed, a series of three initial runs is conducted: one run at current conditions and two runs at perturbations of one or both of the process variables. The resulting process performance is noted and the run which led to the *least favorable* results is identified. A new run is performed at conditions which are the mirror image of the least favorable run. This new run replaces the least favorable run in the simplex. This leads to a new least favorable condition in the simplex which, in turn, leads to another new run, and so on. In this manner, the process eventually moves from the current operating conditions to ones that result in more favorable performance. This is done, at each step, by *moving away* from the conditions which yielded the least favorable results.

Example of simplex EVOP

A process is operating with a scrap rate of 17%. Improved process conditons to reduce this rate are desired. Oven temperature and feed rate are identified as two process variables which, if set differently, might result in such a reduction. These two variables are currently set at 200° and 30 units, respectively. An increase of 10° in temperature and 2 units in feed rate might lead to some improvement in the scrap rate and is unlikely to result in any radical deterioration. From this information, the conditions for three initial runs, forming the corners of triangle A (Figure 1), are defined:

Run No.	Temp	Feed rate	Resulting %scrap
1	200	30	17.2
2	210	30	16.2
3	205	32	16.6

Run 1 is at current operating conditions. Runs 2 and 3 involve perturbations away from the current operating conditions, involving increases up to 10° in temperature and up to 2 units in feed rate. The nature of the operation makes it inconvenient to vary temperature and feed rate during normal production. However, the process is shut down every half day for routine maintenance and for addition of new material. When the process is started up again a different temperature and feed rate can be used. Thus each run involves a half-day of operation.

The three runs are conducted and the percent of scrap on each is recorded. As can be seen from the preceding tabulation, the least favorable result, 17.2% scrap, is obtained on Run 1. The simplex procedure calls for conducting the next run (Run 4) at a condition which is the reflection, or mirror image, of the worst condition away from the current triangle (B) of conditions—a temperature of 215°C and a feed rate of 32. In the new triangle of conditions, Runs 2 and 3 are retained but Run 4 replaces Run 1; i.e., the current triangle of conditions is now:

Run No.	Temp	Feed rate	Resulting %scrap
2	210	30	16.2
3	205	32	16.6
4	215	32	15.4

Run 4 is then conducted and it is noted that Run 3 is now the one with

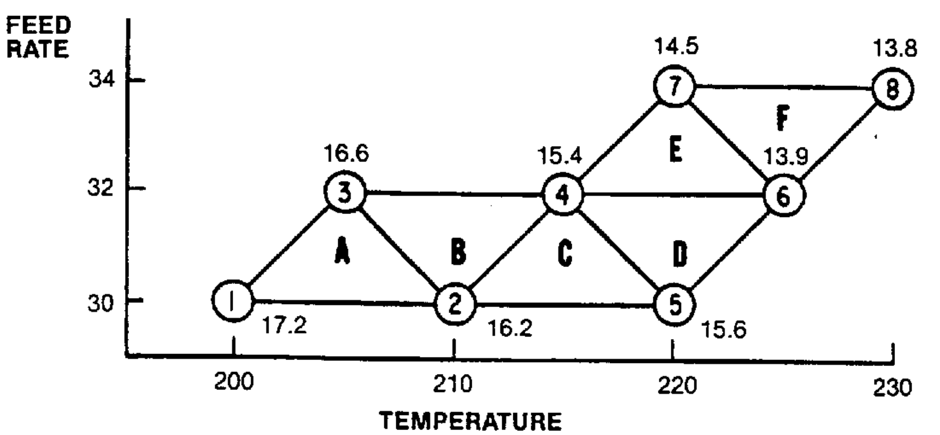

Figure 1. Simplex EVOP Runs 1-8 and resulting percent scrap

the highest scrap. It is replaced by Run 5, the reflection of Run 3 in the current triangle—a temperature of 220° and a feed rate of 30 units. This leads to triangle C:

Run No.	Temp	Feed rate	Resulting %scrap
2	210	30	16.2
4	215	32	15.4
5	220	30	15.6

Run 2 is now the run in the current triangle with the highest percent scrap. Therefore, the next run is conducted as the reflection of Run 2.

The general procedure is now evident. After each run, a new triangle of conditions is defined by replacing the run in the current triangle that yielded the least favorable results ("worst run") with a new run, which is its reflection away from the current triangle. The other two runs (OK Runs 1 and 2) are retained in the current triangle. The conditions at which the new run is to be conducted can be determined graphically or from the following simple expressions:

- Value for process variable 1 for new run = (value of process variable 1 for OK Run 1) + (value of process variable 1 for OK Run 2)—(value of process variable 1 for worst run).
- Value for process variable 2 for new run = (value for process variable 2 for OK Run 1) + (value of process variable 2 for OK Run 2) — (value of process variable 2 for worst run).

For example, the conditions for Run 6 are: Temperature = (temperature for Run 4) + (Temperature for Run 5) — (temperature for Run 2) = 215 + 220 - 210 = 225°. Feed rate = (feed rate for Run 4) + (feed rate for Run 5) — (feed rate for Run 2) = 32 + 30 - 30 = 32. Proceeding graphically or using the above formula, the first EVOP runs and their results are shown in Table 1.

In this example, the scrap rate was reduced from 17.2 to 13.8% in 8 runs. Needless to say, improvement is not always that rapid, especially if there is much random variation in the scrap rate. However, if the right process variables are selected, the procedure should eventually result in improvements, although chance fluctuations leading to a move in the wrong direction could result in some temporary setbacks.

Process characteristics favoring use of EVOP

The preceding example illustrates the following process characteristics favorable to the use of EVOP:

- There is room for process performance improvement (reduction of the 17% scrap rate)
- Process variables which affect product performance can be identified (temperature and feed rate)
- The identified process variables can be perturbed readily (but, in this example, only each half day).
- The process stabilizes rapidly after a change in conditions (generally so for batch processes, but not always for continuous processes).
- Feedback of performance is rapid (the scrap rate from the previous run is known before the next run has to be started).

The fact that the process conditions can be changed only each half day, rather than more frequently, and the desire for rapid improvement were reasons why simplex EVOP, rather than classical EVOP, was chosen. Simp[illegible] EVOP is also applicable if, in additi[illegible] to minimizing the scrap rate, other performance criteria are to be considered. All that is needed is a determination of the conditions in each triangle which led to the *least favorable* results, so that one can move away from these conditions on the next run.

Simplex EVOP is especially applicable when process performance is changing with time because of variability in material or operating conditions or for other reasons. The rate of change must, however, not be more rapid than the EVOP scheme's capability to react.

Another application is to processes in which a fresh optimization is required for *each* new lot of material. Here simplex EVOP can be used on two levels: (1) to determine the process conditions to be used for each lot, (2) to improve starting conditions in seeking the optimum.

Table 1. An EVOP experiment

Run no.	Temp.	Feed rate	Resulting % scrap	Runs in current triangle	Worst run in current triangle
1	200	30	17.2		
2	210		30	16.2	
3	205	32	16.6	1,2,3	1
4	215	32	15.4	2,3,4	3
5	220	30	15.6	2,4,5	2
6	225	32	13.9	4,5,6	
7	220	34	14.5	4,6,7	4
8	230	34	13.8	6,7,8	7

Table 2.Construction of a multivariable EVOP simplex
Process variable

Run no.	1	2	3	...	*k*
1	c_1	c_2	c_3	...	c_k
2	$c_1 + p_1$	c_2	c_3	...	c_k
3	$c_1 + p_1/2$	$c_2 + p_2$	c_3	...	c_k
4	$c_1 + p_1/2$	$c_2 + p_2/2$	$c_3 + p_3$	...	c_k
.	.	.	.	...	c_k
.	.	.	.	...	.
.	.	.	.	...	.
k + 1	$c_1 + p_1/2$	$c_2 + p_2/2$	$c_3 + p_3/2$	...	$c_k + p_k$

Three or more process variables

Simplex EVOP can be readily extended to situations where three or more potentially significant process variables are to be perturbed. (In fact, the more variables there are, the more likely it is that the simplex approach, rather than the classical EVOP approach, is appropriate.) The triangle for two process variables is then replaced by a tetrahedron for three process variables and its multi-dimensional analog for four or more process variables. . If there are *k* process variables, the current figure is a simplex of ($k + 1$) points. If $c_1, c_2 \ldots c_k$ and $p_1, p_2 \ldots p_k$ are the initial conditions and the allowable initial perturbations, respectively, for the *k* process variables, the conditions for the initial ($k + 1$) runs are as shown in Table 2.

In the example presented earlier, $k = 2$, the initial temperature and feed rate were $c_1 = 200°$ and $c_2 = 30$ units, respectively, and the initial perturbations for these variables were $p_1 = 1$-degrees and $p_2 = 2$ units.

From the results of the initial $k + 1$ runs, the worst run is identified, the remaining runs being denoted as OK runs. A new run is then conducted at conditions which are the reflection away from the worst run and are obtained from the following expression for each process variable:

Value of process variable for new run = 2 (average value of process variable for *k* OK runs) — (value of process variable for worst run). The new run then replaces the worst run in the current simplex, and the procedure is continued.

Some modifications

To avoid difficulties in using simplex EVOP, the following modifications are useful:

1. Rerun any point which has been included in each of the last $k + 1$ (or other prespecified number) simplices. This procedure permits removal from the current simplex of a point which due to chance variations is "too good".

2. If the worst condition is the one which has been added most recently to the simplex, instead of replacing it, replace the second worst condition by its reflection. This avoids returning to the previously poorest condition.

In some situations it might also be reasonable to change the perturbation step size. For example, a larger movement away from the worst condition might be appropriate if that condition has yielded appreciably poorer results than the other conditions in the simplex. Also as an optimum is approached, smaller step sizes might be desirable *(10, 11)*.

Further information on simplex EVOP

My last column provided an introduction to the EVOP concept and to classical EVOP and also included some references. Simplex EVOP was originally introduced by Spendley et al, in Reference 2; References 3 to 9 describe applications and further discussions of this approach. Also a refinement of the simplex method has been applied to mathematical minimization problems *(10, 11)*.

Concluding remarks

This column has dealt with a dynamic form of EVOP known as the simplex method. Unlike classical EVOP, simplex EVOP involves a move to new conditions after each run. Thus this approach is especially appropriate when conditions can be varied relatively infrequently. On the other hand, simplex EVOP generally provides less "scientific insight" than does classical EVOP, and a brief mistaken excursion into a less desirable (rather than a more desirable) region may be more likely. For a more detailed comparison of the two approaches, see Reference 12. In practice, the appropriate EVOP procedure should be tailor-made to the process and operating environment; this may result in a variation or combination of the two methods.

Be it classical or simplex, evolutionary operation involves a *planned* approach for introducing perturbations into the operation of a manufacturing process. This is in contrast to the case where one only observes the normal fluctuations of the process variables and relates these fluctuations to product performance, using least squares regression analysis. Although this procedure might sometimes lead to useful results, it is unlikely to provide unambiguous information for process improvement *(13)* and should not be confused with EVOP.

I have tried to show how evolutionary operation can be used for process improvement. Admittedly, EVOP is not suitable for all processes. However, when a process which involves high-volume production is not operating at its optimum, the merits of using EVOP should be evaluated by the responsible manufacturing engineer.

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