BASIC TOOLS

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NORMAL PROBABILITY PLOTS

INTRODUCTION

Plotting sample data points on probability paper can be a powerful tool in the hands of quality practitioners. Probability paper is specially ruled graph paper with one "regular" axis, and another axis ruled in units of percent probability for an appropriate distribution. The probability ruling is based on a cumulative distribution function. Keuffel and Esser graph paper provides the probability ruling on the x axis and the "regular" ruling on the y. On the other hand, my computer software uses the y axis for the probability ruling.

It is not hard to perform probability plots by hand, as we shall soon see. Fortunately, many computer software packages now include probability plot options, thus improving accuracy and speed allowing us to focus on interpretation rather than mere mechanics.

Depending on the resultant shape of the graph, data points give clues as to the nature of the underlying distribution from which the data came. The idea behind these plots is that the closer plotted points are to a straight line, the more likely it is that they come from the distribution modeled by the probability paper. Among the most widely used probability papers is normal probability paper. With normal probability paper if the data plot close to a straight line, they can be considered to be reasonably Gaussian (normally distributed).

In addition to indicating general distribution shape, probability plots also uncover gaps, clusters, outliers, and multiple modes. It is also possible to obtain approximate sample statistics such as the mean and standard deviation, and to predict percentages expected beyond certain limits. A detailed treatment of the subject is given by King (1971), and simple illustrations of the use of normal probability plots are found in, Heyes (1983), Ott (1975), and many other sources.

HOW TO CONSTRUCT A NORMAL PROBABILITY PLOT

1. Obtain the data of interest.
2. Order the values from smallest to largest.
3. For each data point obtain a probability axis location using * Cunnanes formula; location = 100 * [(i - 3/8) / (n + 1/4)] where i is the ith ordered value in a sample of size n.
4. Plot each point at the appropriate x, y location based on its value for the "regular" axis and the probability location above for the other axis.
5. Draw a straight line roughly dividing the points into two equal halves.

* Other formulas for normal probability location exist including; 100%(i - .5) / n, and i(100% / n + 1), but they yield less accurate estimates of the standard deviation for samples of 20 or less.

EXAMPLE

A number of electronic measurements were taken on 17 prototype video heads. Based on these measurements Engineering wanted to know what kind of performance to expect from future video heads made by the same company.

Table 1 shows the raw data (RF signal level), the ordered data (ascending), and the y-axis position for each ordered data point based upon; location = 100 * [(i - 3/8) / (n + 1/4)], where i are values 1 through 17 and n is fixed at 17.

TABLE 1 — Preparing Data For A Probability Plot

POSITION (i)	Raw Data RF Signal Level mV	Ordered For x-axis	y -axis Probability Position
1	540	520	3.62
2	550	525	9.42
3	530	530	15.22
4	560	530	21.01
5	540	540	26.81
6	560	540	32.61
7	580	550	38.41
8	580	550	44.20
9	560	560	50.00
10	525	560	55.80
11	570	560	61.59
12	600	560	67.39
13	590	570	73.19
14	560	580	78.99
15	530	580	84.78
16	550	590	90.58
17	520	600	96.38

From Table 1 the points would plot as:

1st point at x =520 and y =3.6 = 100 * [(1 - 3/8)/(17 + 1/4)]
2nd point at x =525 and y =9.4 = 100 * [(2 - 3/8)/(17 + 1/4)],
3rd point at x =530 and y =15.2 = 100 * [(3 - 3/8)/(17 + 1/4)].

We continue plotting in this manner until the last point is located at x =600 and y =96.3. The resulting probability plot is shown in Figure 1 (see next page).

It is evident from Figure 1 that the parameter Signal Level is close to normal because of how well the points follow the line. These data are so well behaved that we can make even more use of the probability plot. First we can estimate the mean by locating the x value corresponding to the 50% probability (y) level. This value of 555 mV is shown in Figure 2 (see next page).

Also shown in Figure 2 are x values of 530 and 580 mV corresponding to y values of 16 and 84% respectively. The difference between these two x values divided by 2 gives and estimate of the standard deviation. Our estimated standard deviation for video signal level is:

(580-530) / 2 =25.

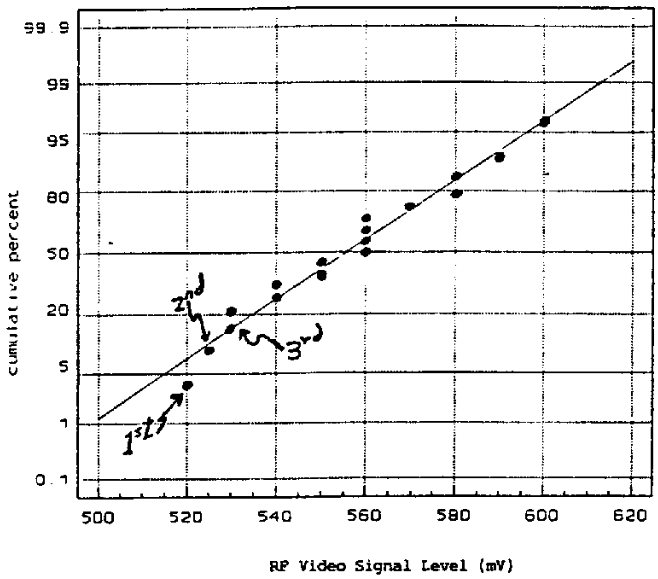

Figure 1 — Normal Probability Plot

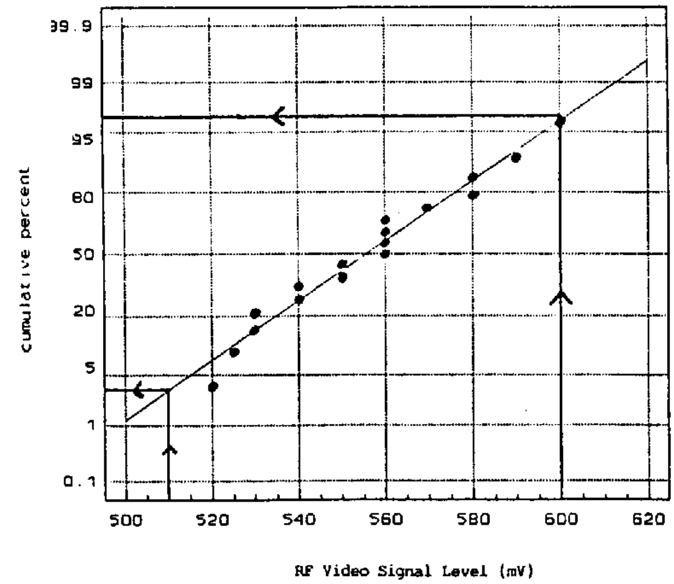

Figure 3 — Percent Beyond Limits

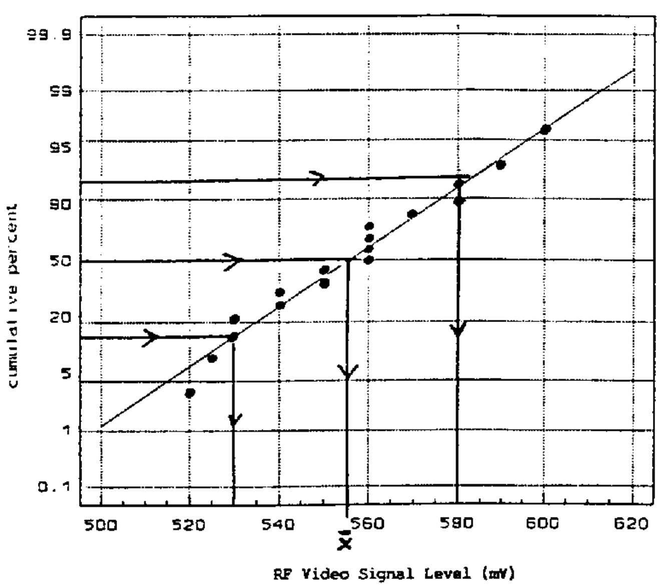

Figure 2— Estimating Statistics

Now suppose engineering wanted to predict what percent would be expected to be beyond proposed specification limits of 510 to 600 mV. To estimate these percentages follow each x value up until you reach the best-fit line, then read across to the corresponding y value. Figure 3 demonstrates this practice, resulting in expected percents of approximately 4% below 510 mV and 4% above 600 (96% at or below 600). This gives a total of approximately 8% beyond proposed specifications, clearly an unacceptable rate.

References

King, J. 1971. "Probability Charts for Decision Making", Team, Tamsworth, N.H.

Heyes, G.B. 1983. "Painless Plots" Quality, November 1983.

Ott, E. 1975 "Process Quality Control" McGraw-Hill, New York. Δ

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