CHAIR’S MESSAGE
by Conrad Fung

How was your summer? I served on a jury for a rather long trial. It was a gratifying experience. Statistical principles were in action everywhere, from my being randomly selected by a computer, to the exhaustive questioning of expert witnesses on whether events in the case were surprising or commonplace (you got it—special causes or common causes). While locked up for deliberation, we used graphical displays on the blackboard to understand the variation in our opinions. The graphs showed at a glance where every juror was coming from, and helped us ultimately to reach agreement.

Statistics and due process go together. Understanding variation and common causes is crucial to responsible decision-making. Even the question of “reasonable doubt” raises a statistical issue—is it one in a hundred? One in a thousand? A hat tip to John Gurland at UW for asking this.

Statistical thinking is natural! Everyone does it all the time in everyday life. So why does it seem so novel to apply it in our jobs? I think my jury experience holds part of an answer to this. Intentionally chosen at random to assure a diversity of backgrounds, our group naturally had a diversity of personalities—including a handful of dominant executive types, and a few who were hesitant in the extreme to offer any opinion at all. But an incredible transformation took place in the jury room. Everyone embraced statistical approaches to summarizing and displaying information (of course we didn’t call it statistics—the word carries a lot of baggage, as Ron Snee says). The outspoken characters accepted a round-robin style of discussion, and the unassertive members became equally as vocal as the others.

Democracy worked in the sheltered world of our jury room, I am convinced, because of a fortunate conjunction of two essential things: (1) we were equally empowered, and (2) we had data. Legitimized by an external authority to take responsibility, even the quietest members said their piece. Armed with no more power than anyone else, the dominant characters managed to set aside personal agendas and acknowledged the higher task of making a responsible decision. Facing down an endless parade of graphs on the blackboard, we worked from facts rather than emotion. Data as equalizer! What a concept.

As to why statistical thinking doesn’t happen more often in our jobs, a quote from one of my students’ homework papers explains a lot (and fits my jury theory): “I talked to an engineer who works for a major manufacturer that claims to be using the Total Quality approach. The engineer was told by his manager that he relied too much on numbers, and needed to use his gut instinct more often.” The requirement for equal empowerment went out the window in this incident, and the data went out with it. Another student’s remark is telling too: “It is sometimes amazing how slowly such critical issues spread through American society. I have no doubt that 80% of all Americans could identify Bart Simpson, but who is this Deming fellow?” Such wisdom from young students. Thanks to John Hallinan and Dave Hasseler, respectively.

My time on the jury was a life-affirming experience. Maybe our group was lucky. But I have come away doubly impressed by the power of collective decision making.

In the Statistics Division Council, we endeavor to work collectively as much as possible, with as much input from Division members as we can get. The membership survey mentioned in the summer Newsletter is now well underway. Its outcomes will provide vital direction to our future efforts. We’ll publish the results in the next Newsletter.

I want to close by thanking a few Division members who work tremendously hard throughout the year to organize sessions at the Annual Quality Congress and the Fall Technical Conference, our main “in-person” opportunities to serve the Society. Rick Schleusener did a yeoman’s job as the Division’s 1991 AQc Program Representative. Greg Gruska has been working hard all summer to organize the Division’s 1992 AQc sessions. Andy Palm’s sure hand helped forge 1991’s excellent FTC program. And I want to welcome Ralph St. John as Statistics Division’s program representative for the 1992 and 1993 FTC’s.

All of our officers, council members, regional councilors, and committee members are volunteers who commit tremendous personal energy to the success of the Division. As ever, I am grateful for your help.
Dear Ms. Baxter,

We are writing in regard to the mini paper "STOP LIGHT CONTROL - Revisited" by Gruska and Heaphy that appeared in the summer issue of the Statistics Division Newsletter. We feel that this paper has no place in a newsletter whose mission statement is to foster "effective use of statistical methods for quality and productivity".

This paper represents a tool that is outdated and long since abandoned in favor of the more appropriate technology of control charts. Let us examine just a few of the misconceptions within this paper.

First, the automobile companies do not want control methods based on tolerances (specification limits). Attribute control charts are derived from statistical distributions. The automobile companies insist on control limits based on statistical data and NOT engineering specifications. This has nothing to do with whether the data is of a variable or attribute nature. Statistical limits are a prerequisite for distinguishing between common and special cause variation.

Second, the example illustrating the process distribution with Cpk = 2 shows warning and stop limits for areas based on the distribution. A value that occurs in the warning area or even the stop area is NOT any indication of a special cause. These ideas and concepts are highly misleading and will lead to many type I errors (tampering).

Finally, the use of double sampling procedures reinforce the concept of inspection tables (in a zone or out of a zone). Drawing two pieces of out of five in the green zone is luck not statistical control.

These three examples do not cover all of the misconceptions in this article, but should have been sufficient to have curtailed its publication in this newsletter. In fact, we see no useful application of the stoplight concept and any benefit is far outweighed by the dangers.

Eileen Beachell
Statistical Consultant
Marilyn Monda
Statistical Consultant

RESPONSE:
Gregory Gruska and Maureen Heaphy

The writers of the letter have two major concerns: (1) the appropriate-ness of material for this newsletter; and (2) technical concepts addressed in the paper. Let us discuss each of these separately.

1. The Statistics Division Newsletter is a major communications channel within the division. Since the majority of Statistics Division members are unable to attend division meetings and functions, the newsletter keeps them apprised of what the Division and its officers and members are doing. In addition to this, the newsletter also supports the Division's Mission statement (see page 2) by including short articles that "promote statistical thinking...and aid in the professional growth and development of division members." If an article gets members to think a little more about the tools they are using or gain additional insight into statistical theory then the article was successful.

2. Controversy is not shied away from and neither "political correctness" nor popularity a prerequisite. Control charts were considered "old and outdated" tools in the 1960's and 1970's - "How could they know?". This did not stop their resurgence in the 1980's. We need to have an open mind and understand the strengths and weaknesses of all the tools available to us.

3. With the move to base the decision criteria of control methods on the process not the tolerance, many "traditional" uses of statistical

Continued on next page
tools are no longer considered acceptable. The paper took one such tool, stoplight control, and "updated" it to a tool based on process performance. With this change, the technique also becomes a valid indicator of the occurrences of special causes.

In the Stoplight scenario, the probabilities depend on how we select the categories. If the categories are determined as in the paper (1.5s increments from the process average) then the probability of getting a single part form a stable process in the red category is approximately 0.135%. The probability of getting a part in a single yellow category is approximately 6.5% and, by pure chance, we will get a yellow once in 7-8 sampled parts (since we have two yellow categories). The probability of getting two sampled parts in yellow categories is 0.4225% (=.065*.065).

Consequently, a single sampled part in the red category or two or more sampled parts in the yellow category ARE indications that something may have changed - a special cause is present.

(2-2) All sampling plans have both Type I (calling a good process bad) and Type II (calling a bad process good) errors. We could make the Type I error zero by always accepting the process - regardless of our analysis. This unfortunately would drive the Type II error (and Customer DISsatisfaction) sky high. Alternately, we could make the Type II error effectively zero by having restrictive decision rules; but this leads to tampering. The focus should be to balance both of these errors to maximize value to the customer. This is where statisticians and the subject matter expert earn their money.

Traditionally, Type I error rates are taken to be in the 1% -5% range. The Average and Range charts have a very low false alarm rate (Type I) rate - 0.27% - regardless of the sample size. But, they also have poor sensitivity (i.e. high miss rates - Type II error) for small shifts in the mean. It is recommended in the literature that if small shifts (<.5s) need to be detected, alternate approaches such as the Cusum, WA, or EWMA be used in place of the Average and Range charts.

The (2,3) stoplight plan given in the paper has a false alarm rate of 2.38%; higher than the Average and Range charts but still "respectable". Correspondingly, it has better sensitivity to small shifts than an Average and Range chart of sample size 5. The ability to detect large shifts is virtually the same for both plans. Since the risk of making a wrong decision is part of any statistical technique, we must understand what these risks are and how they will impact the total process. It is only through full knowledge and understanding of statistical tools and techniques that we can assist industry, education, and government in the effective use of statistics.

(2-3) Finally, the use of sequential sampling is a technique used to minimize the total number of samples required to make a decision. If our process experienced a large shift, we would not need to measure a large sample to be certain something changed. If our process has not changed (i.e. remained in statistical control) we do not want to measure a lot of part to verify it. Instead we sample sequentially.

We start off by giving the process the benefit of the doubt - assume that it is in statistical control. If our first sample is consistent with this assumption, we say the process is still in statistical control. If we find a sample that is totally inconsistent with this assumption - a sample in the red category - we say we have proof that the process is out of statistical control. If we find a sample that may or may not support our assumption, we take more sample; i.e. get more information. The number of times we go through this cycle depend on economics and the risks we are willing to accept.

In conclusion, to encourage or discourage the use of a technique without understanding, without the benefit of profound knowledge is suboptimal in the long term. The reputation of all statistical tools, control charts included, suffer from their misapplication, regardless of the motivation or intent. As members of the Statistics Division of ASQC we must always strive to "promote (correct) statistical thinking for quality and productivity improvement".

Nancy,
I have been disturbed more than once by the Editor's Corner block. I have underlined the sentence of concern: "Neither I nor the printers are going to sort through over 14,000 label to find yours." Within the quality philosophy we say - satisfy the customer. Sometimes that is not possible. When it is not possible you either say so, find an alternative or you get out of that business.

It appears that you are faced with the 1st alternative - not being able to comply. Fine, but the customer doesn't want to know your problems or your reasons why. I don't appreciate you making us (customer of yours) feel like we're inconvenienced you by asking for an address change. After all (as a previous editor myself) that is part of your job. You do offer an alternative - contact ASQC. That is all that is necessary. Put yourself in the reader's shoes and read your block. It paints a picture of you that I'm sure is not worthy!

My suggestion: drop the sentence and your customers will survive and do the right thing with their address changes.

Lisa Albiz
Lisa,Thanks for the suggestion. As you will notice, the Editor's Corner has been revised.

Dear Ms. Baxter,
This is in response to your "Name the Newsletter Contest" item in the Statistics Division Newsletter, Summer 1991, page 15. There is an unproved assumption here, namely, that the division members want a different name for the newsletter. Where is the data set supporting this idea?

Wouldn't it have made more sense to survey the membership on this point? Perhaps 75 percent would agree with me that the present name is just fine. It appears that the plan is to choose a new name from the responses of those who think that there should be a new name!

However, all is not lost. Why not proceed as follows. Screen the entries to yield the 10(?) best. Then present these to the membership along with the choice "no change in

Continued on next page
Dear Readers:

The mail has been flowing in the last 2 months. I have received over 125 submissions for the "Name the Newsletter Contest". Thanks to all those who sent along ideas or comments about the newsletter. We are just beginning to sort through the entries and plan to reach a decision by the next newsletter.

I have received several submissions for the Mini-paper column but I am still looking for submissions for the Basic Tools column. Is there a specific topic you would like to see presented?

You will notice the "This Quarter in JQT" column has taken on a new look. This column will now feature the table of contents for the latest issue of the Journal of Quality Technology.

As much as I enjoy being editor, I plan to move on to other things at the end of the term (July 1). I have found this position to be rewarding and an excellent opportunity for meeting people within ASQC. We are looking for someone who might be interested in serving as editor. A job description appears elsewhere in the newsletter. Please contact myself or one of the division officers if you are interested.

Nancy

Letters to the Editor

Continued from 3

name", and ask them to vote. You would then be essentially fully protected against people who carp, like

Yours truly,
Lloyd S. Nelson
Director of Statistical Methods
Nashua Corporation

Lloyd, If we choose to keep the current name of the newsletter, we'll send you the $50.

CHANGE OF ADDRESS

Please note the following changes of address:
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Please note the following change of address for ASQC headquarters.
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New Technometrics Editor

Dr. Stephen B. Vardeman will be the next Editor of Technometrics. He will assume the position of Editor-Elect on January 1, 1992.

47TH CONFERENCE ON APPLIED STATISTICS

Statistics Division will sponsor two sessions at the 47th Conference on Applied Statistics. This conference will be held December 16-18, 1991 at the Sands Hotel, Atlantic City, New Jersey. Anyone interested in registration material can contact: Walter R. Young, Medical Research Division, American Cyanamid, Bldg. 60, Room 203, Pearl River, New York, 10965. (914) 732-3224.

Statistics Division Sponsored Sessions

Regression Analysis By Example
Speaker: Professor Samprit Chatterjee, New York University
Principal Component Analysis with Applications in Quality Control
Speaker: Dr. J. Edward Jackson, Consultant, Rochester, NY

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Using any one of many available computer packages, it is possible to analyze data and prepare reports, using graphs to communicate the most important information. Graphs allow readers to understand relationships, data centering and variability very readily. Well-designed graphs contain tremendous amounts of information and our minds interpret such “pictures” very easily. But graphs are not a substitute for judgement and experience; like any other tool they must be understood and used correctly. They are particularly useful in “exploratory data analysis,” where one is in the initial stages of studying a problem, to help plan more formal experiments, etc. This exploratory process is like peeling an onion; as each layer is removed you gain more insight into the process factor(s). Using graphics, the accompanying tears as you peel the onion will be minimized.

Most current texts on statistical methods emphasize the value of using simple graphs and plots to get an overall view of the data, look for errors, outliers, non-normality, etc. We will see how to do this as we proceed.

I have used data from a plastic injection molding process on a characteristic, “Housing,” to illustrate a graphical approach to data analysis and reporting. Our objective is to run experiments to judge the capability of the process and to prepare a report with our conclusions and recommendations. Graphs are particularly useful in this situation because of the complexity of the molding process due to the multiple sources of variation.

The first stage in our plan is to take a sample of 10 complete shots taken over one shift of production. (A “shot” is one piece from each cavity taken during one cycle of the molding press.) Figure 1 shows measurement data from this sample of the housing dimension, displayed as a “Stem-and-Leaf” diagram. (This technique is explained in the summer issue of the Statistics Division Newsletter.) Two tentative conclusions can be drawn; one piece at -.0019 may be an “outlier,” and the data looks distinctly bi-modal. We have peeled one layer from the onion. Figure 2 is a histogram of the same data so you can compare the two graphical approaches. My preference is the Stem and Leaf diagram because it displays every number. The Histogram abbreviates the amount of information by using class intervals based on arbitrary criteria.

Next, let’s use a general purpose tool called “stratification,” in which we break down the data into levels or strata, to obtain a different perspective. Fortunately, the data is classified by time of sampling and mold cavity number, two obvious ways of stratifying molding data.

A simple “Multi-Vari” plot gives a “snapshot” of the differences in dimensions of parts from the 8 cavities and 10 shots as we can see from Figure 3. Further deductions can be made from this chart. The possible outlier is from cavity 4, shot number 9.

Cavity 4 output consistently runs well below the other cavities and cavity 8 is usually the highest. This fact explains the bi-modal appearance of the histogram and the Stem-and-Leaf diagram. Now we have peeled the “onion” down to a more useful level.

It is evident from Figure 3 that the averages of the 8 cavities varied somewhat, but the variability within each cavity was about the same. An idea of the variability over one shift of the process can be derived from this graph.

The dimensional variation of parts produced by a plastic molding process has two primary causes.

1. Variation between cavities, as we have seen in Figure 1. These differences should remain stable, as they are the result of the way the steel in the mold was cut and the mold/process design. This source of variation is visible as the differences in the cavity averages. Thus each cavity is a “mini-process” with it’s own average and variation.

2. Variations from cycle-to-cycle of the molding process. This is the result of many small changes in temperatures, pressures, timing and a whole host of other factors. This source of variation affects all cavities on each cycle, although not always in the same amount, due to possible interactions. This variation can be measured by changes in the averages of complete shots of the 8 cavities.
Figure 4 is a very useful type of graph, helpful in stratification, called a “Box Plot,” used here to show the variability between cavities explained in #1. Each cavity has its own box, based on the 10 samples measured for the cavity. The explanations for the features of this type of plot are as follows:

1. The center line in the box is at the median value, which splits the ordered data values in half.
2. The rectangular box ends are placed at the 25th and the 75th percentile (Q1 and Q3) of the measurements, called the “Interquartile Range” (IQR), thus splitting each half in half again.
3. The two lines from the box, called “whiskers,” extend out to a distance of 1.5 times the IQR beyond the box ends, to the “fence.”
4. Points marked with an “*” are beyond the fences and those marked with an “o” are beyond 3 times the IQR and show probable outliers. A “well-behaved” (normal) distribution would have all points inside the fences.

The Box Plot shows at least two and possibly five outliers, which need further investigation. Since all ten shots had been saved, it was possible to recheck these pieces. Three of the outliers were errors in recording and two were measurement errors. Another layer has been peeled away.

You will note the analysis so far has calculated no averages, standard deviations or other formal statistics. It is my experience that one should always review the raw data from a skeptical viewpoint using graphs, common sense and process knowledge, before one proceeds to more formal methods. There may be errors or other problems that require “purification” of the data before continuing.

Figure 5 is a “Notched” Box Plot of the data after purification. The notches around the medians allow us to make multiple comparisons between cavities. If the notches of any two cavities do not overlap, we can be approximately 95% confident that the medians are different. We still have one outlier “*,” but this one was not a measurement or recording error.

Now we can safely confront a potentially more costly and difficult question. What shall we do about cavities 4 and 8? Both medians are away from the target of -.0004 in., and also from the other cavities.

Let us assume that the decision is to rework these two cavities to bring them closer to target. After the rework, a further sampling of 10 shots is selected over one shift. Figure 6 is a Notched Box Plot that shows the result of this action. Notice that even after reworking cavities 4 and 8 we still have significant differences between cavities, but the range of these differences is much less. Assuming a specification of -.0004 to .0001, the capability of the process may be acceptable.

To gain further assurance, we resampled the process with 10 complete shots randomly taken over a one week period; Figure 7 shows the result. This confirms the earlier sampling. We find no outliers and the notches overlap on all 10 samples. The process appears well centered and capable of meeting customer requirements as reflected in the specifications.

In investigations of this type, it is important to determine if the data can be classified as “normally distributed,” since the use of most statistical techniques requires normality. For instance, if we wanted to make predictions about the capability of the process with probability limits, normality would be required.

Figure 8 is a “Normal Probability Plot” that shows a graphic, approach to an evaluation of the normality of the data from the sample used in Figure 7. A normal probability plot is made as follows:

1. The data is ordered and for each point an associated percentile value is calculated.
2. The data is plotted against the Housing dimension on the Y-axis and the cumulative normal percentiles on the X-axis.
3. Some PC programs convert the percentiles in #1 above to Z values and then plot them against a linear axis of Z values ranging from -3 to +3. The end result is the same in that a straight line fit indicates normality.

Fortunately, PC statistics software does all this heavy labor with a click of a mouse. If a straight line fits the data “reasonably well,” then we judge the data to be normally distributed. From a graphical standpoint, this approach is superior to superimposing a normal curve to a histogram. The eye can judge the fit of the cumulative distribution of the data to a straight line more accurately than the bars of a
I hope our readers will not be too disappointed, but the techniques used here are valid in any case.

More work would have to be done on the measurement system and on "rationalizing" the cavities to center them if we needed to assure normality. Future decisions about the process regarding control procedures and estimates of output capability must consider these factors, so clearly shown by Figure 8.

Figure 7 shows a process which meets customer requirements; Figure 8 says "problem!" What should be done? This is a typical question which must be answered. Graphs can help to illustrate the problems; people must make decisions.

Results of analysis of data from the study of multi-stream processes, such as this example, are difficult to explain in reports. A table of statistics, cavity numbers, percents out of specification, becomes almost overwhelmingly complex, whereas graphs such as Figures 6, 7 and 8 portray the important information very effectively. These three graphs, with appropriate commentary, could form the basis for the final report with conclusions and recommendations.

This brief example has shown only a few of the many powerful graphical techniques available. In recent years, graphical approaches to statistical data analysis and reporting have come to the forefront, spurred on by the many PC software packages that are available to do the "grunt work." I hope all our readers use these techniques to support their analyses.

Reference

STATISTICS DIVISION MEMBERS — OPPORTUNITY TO UPGRADE STATUS TO SENIOR MEMBER

If you are currently an ASQC member who meets the following eligibility requirements you should apply for Senior Member status. Eligible Members must:

1. Be at least 30 years of age, and actively involved in the quality profession for at least 10 years. Graduation in an approved engineering science, mathematics, or statistics curriculum is considered the equivalent of 4 years of professional experience.

and,

2. Qualify under one or more of the following:
   a. be responsible for important engineering or inspection work involving quality control for at least 2 years.
   b. be an instructor of quality control, engineering or statistical methodologies as applied to quality control for at least 2 years; should be capable of teaching a variety of courses in the quality field.
   c. be a professional engineer or member of a technical society of national status in any country for which the qualifications require a standing equivalent to that required for a senior member of ASQC.

All eligible Members interested in upgrading their membership status should contact the Statistics Division Examining Chair, Bob Perry, at Grand Metropolitan Technology Center, 330 University Avenue S.E., Minneapolis, MN 55414, or at (612) 330-8916.

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Quality Press is actively seeking authors who might have insight into quality as it relates the health care industry; transportation; food, drug, and cosmetic industries; retail sales, utilities and other service industries. For more information, contact Deborah Dunlap, Acquisitions Assistant, Quality Press, American Society for Quality Control, P.O. Box 3005, Milwaukee, WI 53201-9488 or call 414-272-8575, extension 7292.

QA DAY AT ARGONNE NATIONAL LABORATORIES
Quality Assurance Day at Argonne National Laboratories will take place in April, 1992. Statistics Division sponsors a session at this conference. Basic tools or applied industrial papers are needed for this session. Presentations should be 45 minutes in length. If you are interested, contact Jed Heyes (708) 291-4229, FAX (708) 291-4280.
The first two articles in the November 1991 issue were presented orally at the Technometrics session of the 35th Annual Fall Technical Conference held in Lexington, Kentucky, October 24-25, 1991.

The first article is “Using Spatial Considerations in the Analysis of Experiments” by Martin O. Grondona and Noel Cressie. Classical experimental design is based on the three concepts of randomization, blocking, and replication. Randomization attempts to neutralize the effects of (spatial) correlation and yields valid tests for the hypothesis of equal treatment effects. More recently, attempts have been made to use the spatial location of treatment effects to improve the efficiencies of estimators of treatment contrasts. This article shows that a simple, flexible spatial-modeling approach to the analysis of industrial experiments (for example, in wafer fabrication) can yield more efficient estimators of the treatment contrasts than the classical approach. The analysis is based on empirical generalized least squares estimation, in which the spatial-dependence parameters are estimated from resistantly detrended response data.

The second article is “Probability Limits on Outgoing Quality for Continuous Sampling Plans” by Lisa M. McShane and Bruce W. Trumbull. Various quality-control sampling plans have been proposed for monitoring continuous production processes. This article considers the continuous plan (CSP-1) developed by Dodge (1943). This plan has been incorporated into MIL-STD-1235B and has found many applications. Various modifications and elaboration of CSP-1 have been proposed, yet the plan remains popular perhaps because of its simplicity. This article investigates the performance of CSP-1 when production run lengths are short or moderate or when the input process is not iid Bernoulli. For finite run lengths, methods are presented for computing upper and lower percentage points (probability limits) for the distributions of the outgoing quality and fraction inspected when the input process is Markov. Both rectifying and nonrectifying inspection are considered. Particular attention is paid to the special case of iid Bernoulli assumptions, including Dodge’s average outgoing quality limit (AOQL) for the iid case, and AOQL Markov input, and unrestricted AOQL values. These limits are proposed as new performance measures, intended to supplement the tables in MIL-STD-1235B for use in selecting CSP-1 plans. The article also discusses the problem of interval estimation for the incoming and outgoing quality from CSP-1 inspection data.

Vijayan N. Nair, Editor

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ARTICLES
- Prediction Intervals for Some Discrete Distributions - Jagdish K. Patel and V.A. Samaranayake
- An Optimal Design of CUSUM Quality Control Charts - F.F. Gan
- SPC in Low-Volume Manufacturing: A Case Study - George F. Koons and Jeffery J. Luner
- SPC Q Charts for a Poisson Parameter \( \lambda \): Short or Long Runs - Charles P. Quesenberry
- Reliability Test Plans for One-Shot Devices Based on Repeated Samples - Lee J. Bain and Max Engelhardt
- Economic Selection of the Mean and Upper Limit for a Canning Problem - Robert L. Schmidt and Phillip E. Pfeifer
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- Grubbs-Type Estimators for Reproducibility Variances in an Interlaboratory Test Study - Tilmann Deutler
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- Additional Critical Values for Multiple Comparisons by Clust. Analysis - Lloyd S. Nelson
SHAPE-FINDER BOX PLOTS

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ABSTRACT

Since it's introduction by Tukey (1977), Exploratory Data Analysis (EDA) has attracted a great deal of attention. The authors propose a new modification to the box plot indicating degree of "peakedness" via an estimate of Kurtosis. This estimate becomes a single line added to the interior of the box. The position of the line relative to the theoretical normal (location of the whiskers at the midpoint), represents degree of Kurtosis.

KEYWORDS:
Exploratory Data Analysis, Box Plot, Shape-Finder Box Plot, Kurtosis, Percentiles.

INTRODUCTION

Among recent trends in statistics, Exploratory Data Analysis (EDA) has attracted a great deal of attention. Tukey (1977) first proposed the use of EDA for visual data displays, and for preliminary evaluation of data prior to subjecting them to formal analyses. In particular, both Stem-and-Leaf plots and Box-and-Whisker plots display the form of a data set, i.e., whether it is skewed, symmetric, unimodal etc. These plots can also help detect clusters, and outliers. For construction and review see Heyes (1985).

The literature records extensive studies on variations of Stem-and-Leaf and Box plots (Ahmed and Aslam 1988, Beckett and Gould 1987, Heyes 1988, Hunter 1988, McGill, Tukey and Larsen 1987). A new box plot modification is proposed here which provides additional information about the shape or "peakedness" of a data set via a graphic indicator of Kurtosis. The authors suggest the name Shape-Finder Box Plot for this modification.

THE SHAPE-FINDER BOX PLOT

A modification of the conventional box-and-whisker plot, the shape-finder box plot is constructed by simply drawing an additional line within the box. The location of this line relative to the center of the box width where the whisker is normally connected, provides a convenient visual indicator of kurtosis.

ESTIMATING KURTOSIS

While there is considerable disagreement in the literature regarding the definition, meaning and interpretation of Kurtosis, the present authors prefer the simple definition of Moors. Moors (1986) argues that kurtosis is a measure of dispersion in the middle of the distribution and concentration of observations in the tail of a distribution. The concentration in the middle of the distribution may be high or low, consequently the tails are sharp or heavy; hence the distri-
bution may be leptokurtic, mesokurtic or platykurtic respectively. Further, the authors have chosen a simple measure of kurtosis which is consistent with other box-plot measures. This measure, $K$, is called the percentile coefficient of kurtosis and is given by:

$$K = \left( \frac{Q_3 - Q_1}{2} \right) / (P_{90} - P_{10})$$

where $Q_1$ and $Q_3$ are lower and upper quartiles, or by

$$K = \frac{HS}{2} / (P_{90} - P_{10})$$

where HS is the Hinge Spread to be discussed later.

$P_{10}$ and $P_{90}$ are the 10th and 90th percentiles respectively of the ordered data. A standard value of $K = 26.3\%$ is obtained for the normal, mesokurtic distribution. Leptokurtic and platykurtic distributions obtain $K$ values less than 26.3% or more than 26.3% respectively, as shown in Figure 1.

COMPUTING QUANTILES

Box Plot Degrees of Kurtosis

To obtain the necessary hinges, percentiles and median the authors recommend using a stem-and-leaf plot as described in example 1.

EXAMPLE 1

Table 1 shows pulse rate measurements from 45 students.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Ordered Leaves</th>
<th>No. Leaves</th>
<th>Cum. Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*60</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>*70</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>.70</td>
<td>5</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>*80</td>
<td>0 0 0 1 1 1 2 3 4 4</td>
<td>13</td>
<td>25</td>
</tr>
<tr>
<td>.80</td>
<td>7 7 8 8 8</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>*90</td>
<td>0 0 0 0 0 1 1 1 2 2 2 2 4 4 4</td>
<td>14</td>
<td>44</td>
</tr>
<tr>
<td>.90</td>
<td>7</td>
<td>1</td>
<td>45</td>
</tr>
</tbody>
</table>

The "depth" of a measure is given by formulas and computed as below. See Freund and Perles 1987 for the $d(P)$ formula.

Sample size $N=45$

$$d(M) = \text{Depth of Median} = (N+1)/2 = 46/2 = 23$$

$$d(H) = \text{Depth of Hinges} = (d(M)+1)/2 = 24/2 = 12$$

$$d(P) = \text{Depth of Percentiles} = 1+((N-1)/10) = 5.4$$

The values corresponding to the depth of the measures are obtained by counting inward from the largest (or smallest) values in the ordered data. For instance the lower Hinge is located by counting in 12 values from and including, the smallest observation. The 12th value of 79 is easily located from the stem-and-leaf plot in Table 2. Table 3 summarizes the depths and corresponding values for the measures in example 1.

TABLE 3

<table>
<thead>
<tr>
<th>Measure</th>
<th>Depth</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median (M)</td>
<td>23</td>
<td>83</td>
</tr>
<tr>
<td>Hinges (H)</td>
<td>12</td>
<td>79, 90</td>
</tr>
<tr>
<td>Min, Max</td>
<td>1</td>
<td>64, 97</td>
</tr>
<tr>
<td>Percentiles (P)</td>
<td>5.4</td>
<td>71.6, 92</td>
</tr>
</tbody>
</table>

BOX PLOT CONSTRUCTION

First draw a simple box plot with length equal to the interquartile range, $IQR = (Q_3 - Q_1)$ or the Hinge Spread HS. The whiskers extend to the highest and lowest values, and location on the x-axis corresponding to the lower whisker location. Next assign an x value of 0 to the side of the box to the left of the whisker. Finally the value of 52.6% is assigned to the x location corresponding to the right side of the
box. By this method each side of the box is 26.3% units from the central whisker. For computed values of K greater than 52.6%, simply continue the scale upwards in major increments of 26.3% truncated at the ending value of 100%.

From table 3, the subsequent calculation of K, and the x-axis scaling, construction of the shape-finder box plot is illustrated in figure 4. In this example, other box plot features (fences, confidence envelopes etc.), have been omitted to draw attention to the location of the K value line.

Figure 4 displays the following obvious properties:

I) The Median value is about 83
   - Location of internal median line relative to X-axis scale
II) There is a slight positive skew
   - Location of median line relative to the hinges (box ends)
   - Relative lengths of the two whiskers- Confirmed via computed coefficient of skewness -0.583
III) There are no outliers in the sample data
   - No values exceeding computed inner/outer fences
IV) The data shape appears to be almost mesokurtic
   - Location of K value line relative to whiskers
   - Shape of the stem-and-leaf plot

REFERENCES

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ADDENDUM
The main idea, that of modifying a box plot with Moor's estimated K value as a line, was presented to Jed Heyes some time ago by co-authors Aslam and Khurshid. Recognizing that the central theme may have some merit, Heyes responded to their request for assistance in developing it thus far. Jed would be delighted to hear from readers who have comments or suggestions for improvement. Some already made include: (1) Determine the operating characteristics of this K statistic relative to the classic B2 measure. (2) Find the optimum scaling for the ends of the box as opposed to the most convenient and determine it's sensitivity to K values. (3) Alternatively, scale the ends of the box based on computed K values for distributions at the extremes of Kurtosis, ie; the most common leptokurtic and platykurtic. (What are these?) (4) Standardize this unusual K value of 26.3% by dividing it by 26.3% thus the standard normal would have a standardized K value of 1.0. (5) How useful is this as a tool?
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